Kelvin-Voigt Analysis of the Stretch or Recoil of Cuticle

Overview

Multiple processes are involved in the stretch of a polymeric material; for proteins this would include, for example, disentanglement of side groups and uncoiling of the protein backbone, all aided by vibrational and rotational motion within the molecules (Ferry, 1980; Vincent, 1990). Viscoelastic materials can be further characterized by their spectrum: the time scales over which stretch occurs. The technique of Tschoegl (1989) is used to analyze the spectrum of cuticular stretch; this method models the observed time-dependent stretch as an initial immediate stretch and a series of exponential functions at distinct time scales with unique individual parameters for viscosity and modulus.

We perform the analysis on compliance, J (MPa^{-1}). The calculation is based on engineering strain (ES), delta L/L_o, at constant (engineering) stress. Compliance is ES/stress, and hence the time dependence of compliance and ES is identical. The analysis proceeds stepwise: Step 1 calculates the contribution of long term processes, by best fitting the curve of stretch to an exponential function in the time range of 1500 to 3000 seconds (400 to 1500 seconds for recoil). The stretch (recoil) associated with this exponential function is calculated and subtracted from the total stretch, leaving a residual. Step 2 then best fits the residual stretch in the range of 200 to 1000 seconds (100 to 250 seconds for recoil); Step 3 20 to 120 seconds (10 to 25 seconds for recoil); Step 4 2 to 10 seconds (2 to 6 seconds). Remaining residual stretch after the four Steps (Step 5) is treated as "instantaneous", and no exponential function is calculated.

Time ranges for the Steps were developed by trial and error with samples of the alloscutal cuticle of *Amblyomma hebraeum*, and might be different for other species/materials. Note that the stretch and recoil is not reversible and symmetric: in stretch a stress is applied, and then suddenly removed in recoil. This may account for the different time ranges between stretch and recoil. In practice the start of stretch or recoil (the placement or removal of a weight on the apparatus) is accurate to only ± 1 second. Hence in practice we lump the modulus of Steps 4 and 5 as a single value.

The exponential for each step is in the form of

$$J = J_{step} * (1 - e^{-t/tau})$$

where tau, the characteristic time of the step, is the viscosity associated with that step, Pa s, divided by the modulus associated with that step, Pa.

Technique

Referring to the worksheet:

1. Enter values from the experiment: mass of weight added or removed, cross sectional area, apparatus correction, and initial length (units are identified in the worksheet). The apparatus correction is the deflection when the weight is added/removed with no cuticle: it is the stretch from the experimental equipment independent of the material, and must be subtracted from the measured stretch. Note that when working with loops of tick cuticle the recoil was measured with 5 g of weight remaining on the loop, to keep it taut.

2. Copy the measured time and distance of stretch/recoil in columns A to C, row 42 and below.

3. Identify the "start" of stretch/recoil from the data in column C, the point at which significant deflection is observed. Cell G78 has the formula "=C79-C76-F51". Change C79 to the first cell at the start of stretch/recoil, and C76 to whatever cell precedes the start of stretch recoil, ensuring that the $ symbols stay in place for the middle value in the equation.

4. Copy the four cells G, H, I and J78 down to row 5077, or the row at the end of the recorded data. At this point the bottom two graphs will have a curve appear. Note that the raw data on recoil will be a declining distance; the recoil calculation worksheet converts the sign so that distance of recoil is treated as a positive number.

5. Adjust cell H55 to maximize the R^2 value for the lower right hand graph. It is important in stretch to exactly maximize this value, since the value of Jinf (the ultimate asymptotic value of compliance predicted by the five step analysis) is sensitive to small changes in R^2.

6. Enter the values for correlation to straight line fit, intercept and slope from the best fit line in cells H56, 57 and 58. The worksheet will then calculated modulus, viscosity, characteristic time tau, and J1 as a % of Jinf. It will also populate cells K, L and M 78, and N49.

7. Maxwell viscosity (the inverse of the slope of compliance vs. time) can be calculated from the equations in the lower left hand graph. Note that one calculation is at 1/3 tau 1, the characteristic

time constant (viscosity/modulus) from Step 1. The value of 1/3 of tau 1 is in cell N49; using "Select Data" for the lower left hand graph, adjust the range of "Maxwell Analysis 33% of Tau" to the value of N49 ± 200.

8. Copy Cells K, L and M 78 down to row 5077, or the row at the end of the recorded data. A second graph will appear on the lower right hand graph. Enter the values for the correlation to straight line fit in cells M 56, 57 and 58.

9. Repeat Step 6 and 7 two more times in columns N, O and P, and Q, R and S.

10. Copy cells T and U 78 down to row 5077, or the row at the end of the recorded data.

At the end of the calculation procedure the worksheet has four graphs. The upper left graph shows total compliance, the contribution of the four exponential functions, and the fit of the calculated model (orange) to the actual recorded value (red). The upper right hand graph shows calculated model vs. actual measured data: a straight line with a slope of 1 would indicate perfect fit. (Because we know that there are some longer term stretch processes that occur at times longer than 3500 seconds, the calculated model values are often slightly less than actual measured values near 5000 seconds.) The lower left graph shows the compliance curve and where the various slopes are calculated for the determination of the Maxwell viscosities. The lower right hand graph shows the best fit lines to model the compliance as four exponential functions and an initial stretch/recoil.

Questions regarding the worksheets and calculation procedure can be addressed to peter.flynn@ualberta.ca .

References

Ferry, J. D. (1980). *Viscoelastic Properties of Polymers*, New York: John Wiley & Sons.

Tschoegl, N. W. (1989). *The Phenomenological Theory of Linear Viscoelastic Behavior*, Berlin: Springer-Verlag.

Vincent, J. F. V. (1990). *Structural Biomechanics Rev. ed*., Princeton, NJ: Princeton Univ. Press.